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MONOTONE ITERATION SCHEME FOR A FORCED DUFFING EQUATION WITH NONLOCAL THREE-POINT CONDITIONS

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ABSTRACT. In this paper, we apply the generalized quasilinearization technique to a forced Duffing equation with three-point mixed nonlinear nonlocal boundary conditions and obtain sequences of upper and lower solutions converging monotonically and quadratically to the unique solution of the problem.

1. Introduction

Duffing equation is a well known nonlinear equation of applied science which is used as a powerful tool to discuss some important practical phenomena such as periodic orbit extraction, nonuniformity caused by an infinite domain, nonlinear mechanical oscillators, etc. Another important application of Duffing equation is in the field of the prediction of diseases. A careful measurement and analysis of a strongly chaotic voice has the potential to serve as an early warning system for more serious chaos and possible onset of disease. This chaos is stimulated with the help of Duffing equation. In fact, the success at analyzing and predicting the onset of chaos in speech and its simulation by equations such as the Duffing equation has enhanced the hope that we might be able to predict the onset of arrhythmia and heart attacks someday. Such predictions are based on the numerical solutions of the Duffing equation. However, there do exist a number of powerful procedures for obtaining approximate solutions of nonlinear problems such as Newton-Raphson method, Galerkins method, expansion methods, iterative techniques, method of upper and lower solutions to name a few. The monotone iterative method and Newton's method are known to be two efficient techniques for finding roots of nonlinear equations. The first one applies to equations involving monotone operators and produces a sequence converging monotonically to a solution. The Newton method has the advantage over the monotone iterative method that it provides quadratically convergent sequences. Applied to nonlinear differential equations, Newton's

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