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On fixed points of quasi-contraction type multifunctions

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ABSTRACT

In 2009, Ilić and Rakočević proved that quasi-contraction maps on normal cone metric spaces have a unique fixed point (Ilić and Rakočević, 2009 [6]). Then, Kadelburg, Radenović and Rakočević generalized their results by considering an additional assumption (Kadelburg et al., 2009 [7]). Also, they proved that quasi-contraction maps on cone metric spaces have the property (P) whenever $\lambda \in (0, \frac{1}{2})$. Later, Haghi, Rezapour and Shahzad proved same results without the additional assumption and for $\lambda \in (0, 1)$ by providing a new technical proof (Rezapour et al., 2010 [4]). In 2011, Wardowski published a paper (Wardowski, 2011 [8]) and tried to test fixed point results for multifunctions on normal cone metric spaces. Of course, he used a special view in his results. Recently, Amini-Harandi proved a result on the existence of fixed points of set-valued quasi-contraction maps in metric spaces by using the technique of Rezapour et al. (2010) [4]. But, like Kadelburg et al. (2009) [7], he could prove it only for $\lambda \in (0, \frac{1}{2})$ (Amini-Harandi (2011) [3]). In this work, we prove again the main result of Amini-Harandi (2011) [3] by using a simple method. Also, we introduce quasi-contraction type multifunctions and show that the main result of Amini-Harandi (2011) [3] holds for quasi-contraction type multifunctions.

1. Introduction

Let (X, d) be a metric space. A set-valued mapping $T : X \to CB(X)$ is said to be a set-valued quasi-contraction whenever there exists $\lambda \in (0, 1)$ such that

$$H(Tx, Ty) \leq \lambda \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$$

for all $x, y \in X$. There have appeared some papers on fixed points of quasi-contractions [1,2]. Recently, Amini-Harandi proved the following result [3] on the existence of fixed points of set-valued quasi-contractions in metric spaces by using the technique of [4].

Theorem 1.1. *Let (X, d) be a complete metric space and $T : X \to CB(X)$ a set-valued quasi-contraction for some $\lambda \in (0, \frac{1}{2})$. Then T has a fixed point.*

He raised immediately the following question.

Question. Does the conclusion of Theorem 1.1 remain true for any $\lambda \in [\frac{1}{2}, 1)$?

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